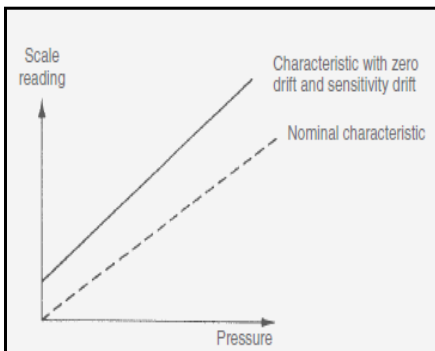


# Instrumentation and Measurements

## ENEE4304

Nasser Ismail

### L3



## Instrument Performance Characteristics

### Static characteristics of instruments

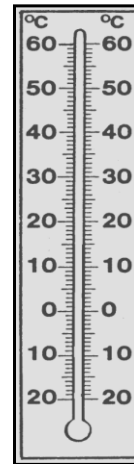
- Range and full scale
- Accuracy/Tolerance
- Precision/repeatability/reproducibility
- Linearity
- Sensitivity of measurement
- Sensitivity to disturbance
- Hysteresis effects
- Dead space/Threshold
- Resolution

## Range and full-scale (f.s.)

- ❑ The range or span defines the minimum and maximum values of a quantity that the instrument is designed to measure.
- ❑ The full-scale (f.s.) of an instrument is defined as the arithmetic difference between the end points (of the range expressed in units of the output variable).

Range  
-20 °C to 60 °C

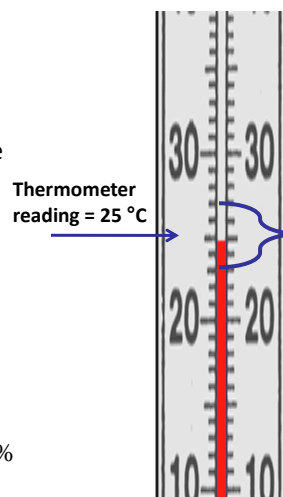
Full scale  
reading (f.s.) =  
 $60 - (-20) = 80$  °C



## Accuracy/Tolerance

- ❑ The accuracy of an *instrument is a measure of how close the output reading of the instrument is to the correct value.*
- ❑ In practice, manufacturers quote the inaccuracy figure rather than the accuracy figure for an instrument.
- ❑ *Inaccuracy is the extent to which a reading might be wrong*, and is often quoted as a percentage of the full-scale (f.s.) reading of an instrument.

$$\text{Inaccuracy} = \frac{\text{Max Error}}{\text{Full scale reading}} \times 100\%$$



If accuracy was quoted as 2.5% in the previous example, then Actual Temperature could be between 23 °C and 27 °C

Since Inaccuracy =  $\frac{2}{80} \times 100\% = 2.5\%$

## Accuracy/Tolerance

❑ **Example 3.1:**

A pressure gauge with a range of 6-10 bar and an inaccuracy of  $\pm 3.0\%$  is used to measure the pressure in a tank. If the reading of the gauge is 7.0 bar, what are the maximum and minimum values of the actual pressure in the tank?

$$\text{Inaccuracy} = \frac{\text{Max Error}}{\text{Full scale reading}} \times 100\%$$

## Accuracy/Tolerance

❑ **Example 3.1:**

A pressure gauge with a range of 6-10 bar and an inaccuracy of  $\pm 3.0\%$  is used to measure the pressure in a tank. If the reading of the gauge is 7.0 bar, what are the maximum and minimum values of the actual pressure in the tank?

**Solution:**

$$\text{Max Error} = (\text{f.s.}) \times \text{inaccuracy} = (10 - 6) \times \pm 3.0\% = \pm 0.12 \text{ bar}$$

$$\text{Maximum value of the actual pressure} = 7.0 + 0.12 = 7.12 \text{ bar}$$

$$\text{Minimum value of the actual pressure} = 7.0 - 0.12 = 6.88 \text{ bar}$$

- ❑ **It is an important system design rule that instruments are chosen such that their range is appropriate to the spread of values being measured, in order that the best possible accuracy is maintained in instrument readings.**

## Accuracy/Tolerance

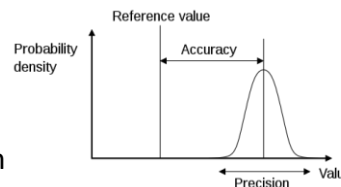
- ❑ Tolerance is a term that is closely related to accuracy and **defines the maximum error that is to be expected in some value**
- ❑ It is commonly used to describe the maximum deviation of *a manufactured component* from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many millimeter.
- ❑ One resistor chosen at random from a batch having a nominal value  $1000\ \Omega$  and tolerance 5% might have an actual value anywhere between  $950\ \Omega$  and  $1050\ \Omega$ .



Maximum allowable diameter : 50.01 mm  
 Minimum allowable diameter: 49.9 mm  
 Tolerance: 0.1 mm

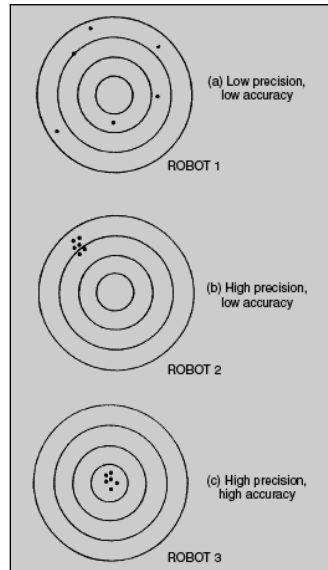
## Precision/repeatability/reproducibility

- ❑ **Precision is a term that describes an instrument's degree of freedom from random errors.** If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small.
- ❑ A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.
- ❑ The degree of repeatability or reproducibility in measurements from an instrument is an alternative way of expressing its precision.



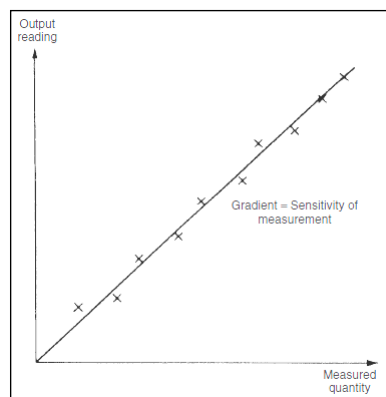
## Precision/repeatability/reproducibility

- Repeatability describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout.
- Reproducibility describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement.



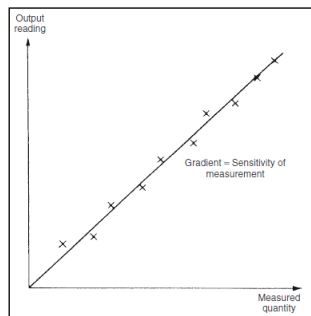
## Linearity

- It is desirable that the output **reading of an instrument is linearly proportional to the quantity being measured.**
- The non-linearity is the **maximum deviation of any of the output readings marked X from this straight line. Non-linearity is usually expressed as a percentage of full-scale reading.**



# Linearity

- If a linear relationship between  $y$  and  $x$  exists for a set of measurements  $y_1 \dots y_n, x_1 \dots x_n$  then this can be expressed as  $y = ax + b$ , where the coefficients  $a$  and  $b$  are constants. Using least squares method the best fit to the measurement data.



$$a = \frac{\sum x_i y_i - n x_m y_m}{\sum x_i^2 - n x_m^2}$$

$$b = y_m - a x_m$$

where  $x_m$  and  $y_m$  are the mean values of  $x$  and  $y$ .

# Linearity

□ **Example 3.2**

The deflection vs. load of a spring scale were recorded as in the table below. Find the non-linearity of the spring scale.

Load (N)	Deflection (mm)
0	0
1	10
2	22
3	28
4	40

	$x_i$	$y_i$	$x_i^2$	$x_i y_i$
	0	0	0	0
	1	10	1	10
	2	22	4	44
	3	28	9	84
	4	40	16	160
sum	10	100	30	298
mean	2	20	6	

$$a = \frac{\sum x_i y_i - n x_m y_m}{\sum x_i^2 - n x_m^2}$$

$$b = y_m - a x_m$$

# Linearity

□ **Example 3.2**

The deflection vs. load of a spring scale were recorded as in the table below. Find the non-linearity of the spring scale.

Load (N)	Deflection (mm)
0	0
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4	40

$$a = \frac{\sum x_i y_i - n x_m y_m}{\sum x_i^2 - n x_m^2} = \frac{298 - 5 \times 2 \times 20}{30 - 5 \times 2^2} = 9.8 \text{ mm/N}$$

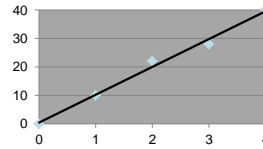
$$b = y_m - a x_m = 20 - 9.8 \times 2 = 0.4 \text{ mm}$$

$$\text{Nonlinearity} = (\text{Max. deviation}) / f.s \times 100\%$$

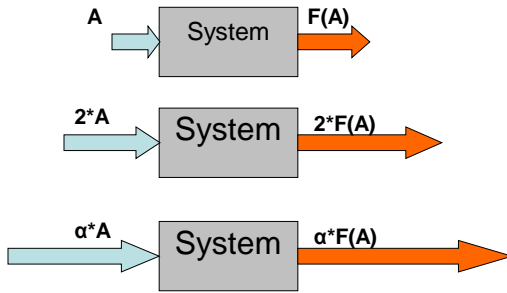
$$\text{Nonlinearity} = (2/40) \times 100 = 5\%$$

xi	yi	xi*xi	xi*yi
0	0	0	0
1	10	1	10
2	22	4	44
3	28	9	84
4	40	16	160
sum	10	100	30
mean	2	20	

xi	yi	axi+b	deviation
0	0	0.4	-0.4
1	10	10.2	-0.2
2	22	20	2
3	28	29.8	-1.8
4	40	39.6	0.4



## What is a linear System?



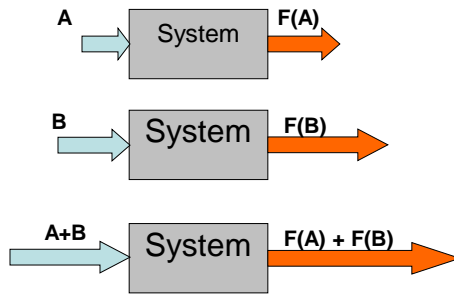
If the Input is doubled, the output is doubled. If the input is multiplied by a scaling factor, the output is multiplied by the same factor.

This is called the scaling property

$$F(\alpha A) = \alpha F(A)$$

**A linear System satisfies the scaling property**

## What is a linear System?

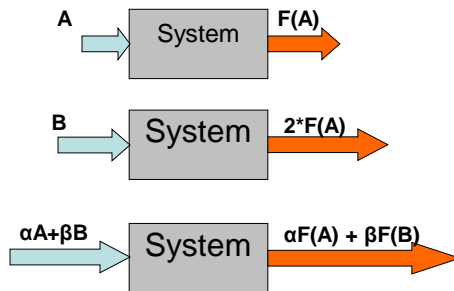


the output to the sum of two or more inputs is the sum of the outputs to each individual input.

This is called the superposition property  
 $F(A+B)=F(A)+F(B)$

**A linear System satisfies the superposition property**

## Linear Systems



In general. A linear system satisfies the scaling and the superposition properties

$$F(\alpha A + \beta B) = \alpha F(A) + \beta F(B)$$



## Resolution

- ❑ When an instrument is showing a particular output reading, there is a **lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output.**
- ❑ The resolution of an instrument influences how finely its output scale is divided into subdivisions.

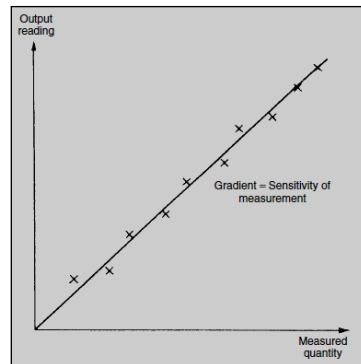


When the needle of a car speedometer is between the scale markings, we cannot estimate speed more accurately than to the nearest 2.5 km/h. This represents the resolution of the instrument.

## Sensitivity

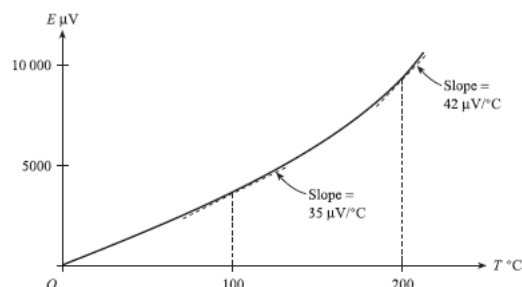
- ❑ The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\begin{aligned} \text{Sensitivity} &= \frac{\text{scale deflection}}{\text{change in measurand producing deflection}} \\ &= \frac{\text{change in output}}{\text{change in input}} = \frac{\Delta O}{\Delta I} \end{aligned}$$



- This is the change  $\Delta O$  in output  $O$  for unit change  $\Delta I$  in input  $I$ , i.e. it is the ratio  $\Delta O/\Delta I$ .
- In the limit that  $\Delta I$  tends to zero, the ratio  $\Delta O/\Delta I$  tends to the **derivative**  $dO/dI$ , which is the rate of change of  $O$  with respect to  $I$ .
- For a linear element  $dO/dI$  is equal to the slope or gradient  $K$  of the straight line;

- Figure shows the e.m.f. versus temperature characteristics  $E(T)$  for a Type T thermocouple
- We see that the gradient and therefore the sensitivity vary with temperature: at  $100^\circ\text{C}$  it is approximately  $35\ \mu\text{V}/^\circ\text{C}$  and at  $200^\circ\text{C}$  approximately  $42\ \mu\text{V}/^\circ\text{C}$ .



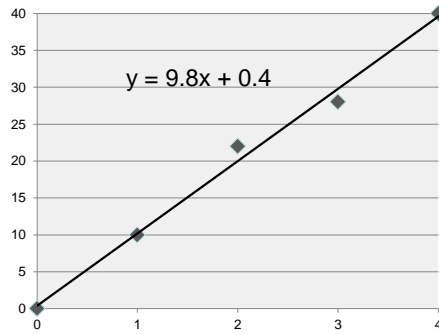
## Sensitivity

### Example 3.3

Find the sensitivity of the spring scale of example 3.2

### Solution

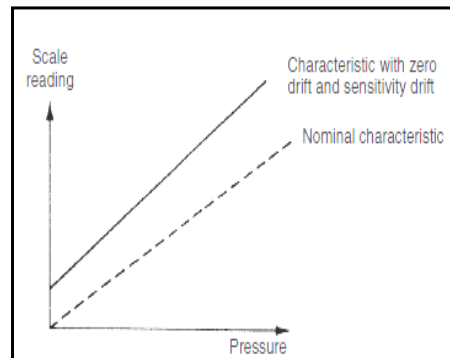
The slope of the best fit straight line was found to be 9.8 mm/N. This is the sensitivity of the spring scale



## Sensitivity to disturbance

As variations occur in temperature, pressure and other environmental conditions, certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change.

- zero drift or bias
- sensitivity drift.



## Sensitivity to disturbance

### Zero drift or Bias

- ❑ **Zero drift** is sometimes known by the term, **bias**. This describes the effect where the zero reading of an instrument is modified by a change in ambient conditions.



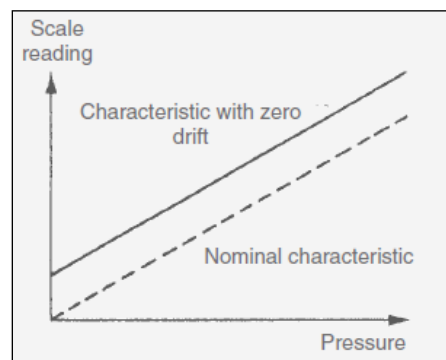
The mechanical bathroom scale is a common example of an instrument that is prone to bias. It is quite usual to find that there is a reading of perhaps 1 kg with no one stood on the scale

- ❑ This causes a constant error that exists over the full range of measurement of the instrument. Zero drift is normally removable by calibration.

## Sensitivity to disturbance

### Zero drift or Bias

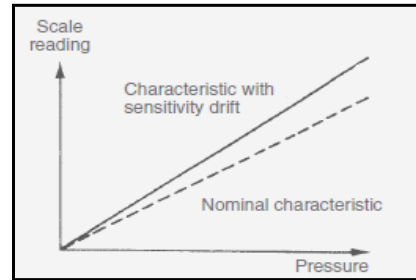
- ❑ Zero drift related to temperature changes is measured in units/°C. This is often called the temperature zero drift coefficient.
- ❑ If the characteristic of an instrument is sensitive to several environmental parameters, then it will have several zero drift coefficients, one for each environmental parameter.



A typical change in the output characteristic of a pressure gauge subject to zero drift

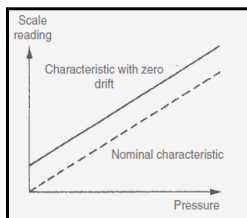
## Sensitivity to disturbance Sensitivity Drift

- ❑ Also known as scale factor drift
- ❑ **Defines the amount by which an instrument's sensitivity of measurement varies as ambient conditions change.**
- ❑ Quantified by sensitivity drift coefficients that define how much drift there is for a unit change in each environmental parameter that the instrument characteristics are sensitive to.

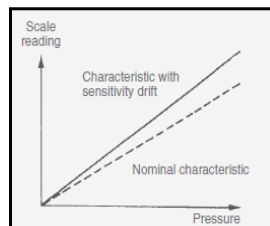


Change in the output characteristic due to sensitivity drift

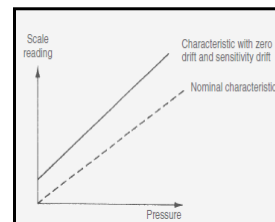
## Sensitivity to disturbance



Change in the output characteristic due to zero drift



Change in the output characteristic due to sensitivity drift



Total change due to zero drift and sensitivity drift

## Sensitivity to disturbance

### □ Example 3.3

A spring balance is calibrated in an environment at a temperature of 25°C and has the following deflection/load characteristic.

Load (N)	0	10	20	30
Deflection (mm)	0	30	60	90

**Determine the zero drift and sensitivity drift per °C change in ambient temperature?**

It is then used in an environment at a temperature of 30°C and the following deflection/load characteristic is measured.

Load (N)	0	10	20	30
Deflection (mm)	7	42	77	112

## Sensitivity to disturbance

### □ Example 3.3

A spring balance is calibrated in an environment at a temperature of 25°C and has the following deflection/load characteristic.

Load (N)	0	10	20	30
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It is then used in an environment at a temperature of 30°C and the following deflection/load characteristic is measured.

Load (N)	0	10	20	30
Deflection (mm)	7	42	77	112

**Determine the zero drift and sensitivity drift per °C change in ambient temperature**

### Solution:

At 25°C, deflection/load characteristic is a straight line.

$$\text{Sensitivity} = 30/10 = 3 \text{ mm/N.}$$

At 30°C, deflection/load characteristic is still a straight line.

$$\text{Sensitivity} = 35/10 = 3.5 \text{ mm/N.}$$

$$\text{Bias (zero drift)} = 7\text{mm}$$

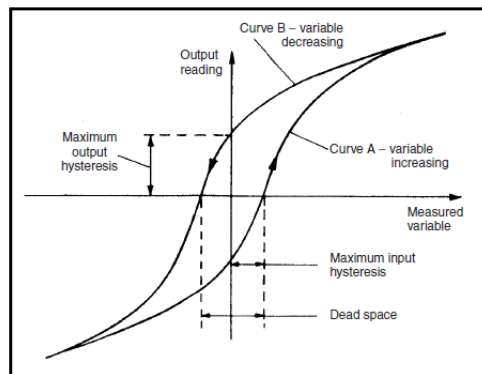
$$\text{Zero drift}/^\circ\text{C} = 7/5 = 1.4 \text{ mm}/^\circ\text{C}$$

$$\text{Sensitivity drift} = 0.5 \text{ mm/N}$$

$$\text{Sensitivity drift}/^\circ\text{C} = 0.5/5 = 0.1 \text{ mm}/\text{N}/^\circ\text{C}$$

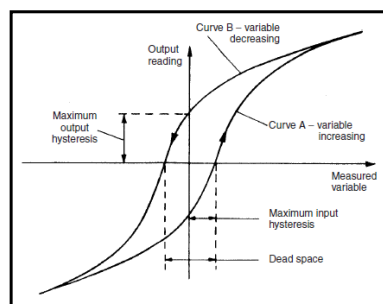
## Hysteresis effects

- ❑ In an instrument with hysteresis effect, non-coincidence between loading and unloading curves exists.
- ❑ Maximum input hysteresis and maximum output hysteresis are defined as shown., These are normally expressed as a percentage of the full-scale input or output reading.



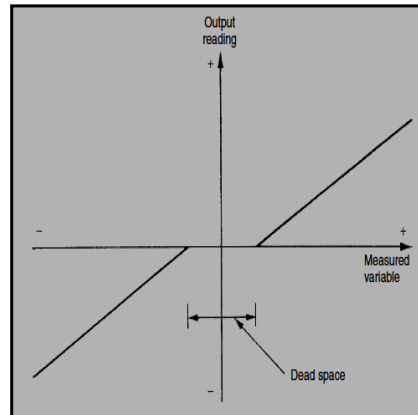
## Hysteresis effects

- ❑ Hysteresis is most commonly found in instruments that contain springs, such as the passive pressure gauge
- ❑ It is also evident when friction forces exist in a system as it have different magnitudes depending on the direction of movement.
- ❑ Hysteresis can also occur in instruments that contain electrical windings formed round an iron core, due to magnetic hysteresis in the iron.



## Dead space/Threshold

- ❑ It is the range of input values over which there is zero output value. Any instrument that exhibits hysteresis also displays dead space.
- ❑ The threshold may also be defined as the minimum value of the input that produces a non-zero output.
- ❑ Some instruments that do not suffer from any significant hysteresis can still exhibit a dead space in their output characteristics.
- ❑ Backlash in gears is a typical cause of dead space. Backlash is commonly experienced in gearsets used to convert between translational and rotational motion.



## Dead space/Threshold

- ❑ For an instrument with dead space, if the input to an is gradually increased from zero, it will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable.
- ❑ **This minimum level of input is also known as the threshold of the instrument.**



**Eddy current speedometers used in automobiles typically have a threshold of about 15 km/h.**